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A Kinetic Derivation of Extended Irreversible Thermodynamics

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The basic postulates of the extended irreversible thermodynamics are derived from the kinetic model for a dilute monoatomic gas. Using the Grad 13-moment method to solve the full nonlinear Boltzmann equation for molecules conceived as soft spheres we obtain the microscopic expressions for the entropy flux, the entropy production, and the generalized Pfaffian for the extended definition of entropy as required by such a theory. Some of the physical implications of these results are discussed.

KEY WORDS: Extended irreversible thermodynamics; relaxation times; constitutive equations; conserved and nonconserved variables; entropy balance equation Boltzmann equation; moment method; Burnett equations.

1. INTRODUCTION

Linear irreversible thermodynamics is based, among other things, on the so-called linear constitutive laws which couple the fluxes in a system with the thermodynamic forces. Examples of these equations are Fourier's law for heat conduction, Fick's law for diffusion, Ohm's law for the conduction of electrical charges, etc.

The validity of equations of this type was already questioned by Maxwell about 116 years ago in his famous paper "On the Dynamical Theory of Gases".(1) There, discussing the concept of viscosity, he pointed out that if in a viscous system under a distortion or strain that excites an elastic force, say, $\mathfrak{T}$ this latter quantity will not remain constantly proportional to the rate of distortion but will disappear at a rate which will depend

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on itself and on the nature of the body. If the rate is proportional to $\mathfrak{T}$ he arrives at an equation which for constant strain reads,

$$\mathfrak{T} = \eta \times \text{const} \exp\left(-\frac{t}{\theta_\eta}\right) \tag{1}$$

where θ_η is the viscous relaxation time and η the viscosity. Thus if the body is left to itself $\mathfrak{T}$ gradually disappears owing to losses in internal stresses, and the pressure is finally distributed as in a fluid at rest. Finally, Maxwell notices that although for fluids θ_η may be very small, of orders of fraction of a second in viscous solids it may of the order of days. Later in the paper using the kinetic model for a gas composed by soft spheres interacting among themselves via a repulsive potential which is proportional to r^{-5} he computes θ_η for air at a temperature of 16°C and a pressure of 1 atm, finding that

$$\theta_\eta = 1.96 \times 10^{-10}\ \text{sec}$$

which is indeed a very small number of the order of magnitude of a mean free time.

These observations of Maxwell were not given their due importance by physicists until 1958 when Cattaneo$^{(2)}$ revived the idea that the linear equations of irreversible thermodynamics ought to be considered as particular cases of relaxation-type equations, so for instance, the traceless stress tensor $\mathring{\mathfrak{T}}$ satisfies the differential equation

$$\frac{d\mathring{\mathfrak{T}}}{dt} = \frac{1}{\theta_\eta}\left[\mathring{\mathfrak{T}} + \eta(\text{gr}\mathring{\text{a}}\text{d}\,\mathbf{u})^s + \cdots\right] \tag{2}$$

which to first order in the gradient of the macroscopic velocity $\mathbf{u}$ leads immediately to Eq. (1). These types of equations are now referred to in the literature as the Maxwell–Cattaneo equations. Clearly, if $\theta_\eta \simeq 0$ or the system is in a stationary state of deformation so that $d\mathring{\mathfrak{T}}/dt = 0$

$$\mathring{\mathfrak{T}} = -\eta(\text{gr}\mathring{\text{a}}\text{d}\,\mathbf{u})^s \tag{3}$$

which is the ordinary constitutive equation for the momentum flow of irreversible thermodynamics.

The extended irreversible thermodynamics (EIT) is a theory which has been developed in the past 15 years with the aim of extending ordinary linear irreversible thermodynamics (LIT) to include the class of phenomena for which the linear constitutive equations such as Eq. (3) have to be substituted by the Maxwell–Cattaneo type of equations. Or to express it differently one desires to build up a theory which can explain nonequilibrium states with relaxation times which are no longer negligible with, say, the time of duration of a certain measurement. This will be the subject of the following section.

2. THE EXTENDED IRREVERSIBLE THERMODYNAMICS

The first attempt to construct this thory is due to Muller,(3) who proposed to modify the local equilibrium assumption of LIT by introducing a dependence of the entropy with additional variables, namely, the heat flux $\mathbf{q}$, the traceless part of the stress tensor $\mathring{\mathfrak{T}}$, and its trace $\mathcal{T}$, which of course disappear when the system is in an equilibrium state. This idea has been taken by a large number of workers(4,5) and given rise to the present conception of EIT, whose basic postulates we shall now discuss.

The first assumption on which the theory rests is related to the variables chosen to describe the state of the system. For a fluid they are the usual local variables, the mass density $\rho(\mathbf{r}, t)$, the macroscopic velocity $\mathbf{u}(\mathbf{r}, t)$ and the internal energy density $e(\mathbf{r}, t)$, plus a set of nonconserved quantities which in the case of the fluid are taken to be the fluxes $\mathbf{q}, \mathring{\mathfrak{T}}, \mathcal{T}$ defined above. Recall that the total stress tensor $\mathfrak{T}$ is defined as

$$\mathfrak{T} = p\mathbf{I} + \mathfrak{T}^v = p\mathbf{I} + \mathring{\mathfrak{T}} + \mathcal{T}\mathbf{I} \tag{4}$$

where p is the local pressure, $\mathfrak{T}^v$ the viscous tensor, $\mathring{\mathfrak{T}}$ its traceless part, and $\mathcal{T} = (1/3)\mathrm{tr}\,\mathfrak{T}^v$, $\mathbf{I}$ being the unit tensor.

This assumption implies that the space of state variables which we shall denote by $\mathcal{G}$, consists of two subsets, one $\mathcal{C}$ formed by the usual conserved densities and another one $\mathfrak{R}$ formed by the additional nonconserved quantities, so that $\mathcal{G} = \mathcal{C} \cup \mathfrak{R}$.

The second assumption is that the entropy S of the system exists and is continuous and differentiable such that its Pfaffian may be written as

$$T\,dS = de + p\,dv + v\boldsymbol{\alpha}_1 \cdot d\mathbf{q} + v\alpha_0\,d\mathcal{T} + v\mathring{\boldsymbol{\alpha}}_2 : d\mathring{\mathfrak{T}} \tag{5}$$

where α_0, $\boldsymbol{\alpha}_1$ and $\mathring{\boldsymbol{\alpha}}_2$ are isotropic tensors of order zero, one, and two, respectively, and $v = \rho_0^{-1}$, ρ_0 being the density per unit mass. As it has been discussed in many papers in the literature,(5)–(7)

$$\begin{aligned}
\boldsymbol{\alpha}_1 &= \alpha_{10}\mathbf{q} + \alpha_{11}\mathring{\mathfrak{T}} \cdot \mathbf{q} + \cdots \\
\mathring{\boldsymbol{\alpha}}_2 &= \alpha_{21}\mathring{\mathfrak{T}} + \alpha_{22}(\mathring{\mathbf{q}\mathbf{q}})^s + \cdots \\
\alpha_0 &= \alpha_{00}\mathcal{T} + \alpha_{01}\mathcal{T}^2 + \alpha_{02}\mathbf{q} \cdot \mathbf{q} + \cdots
\end{aligned} \tag{6}$$

where the coefficients α_{ij} are functions of e, ρ_0, and the scalar invariants of $\mathcal{G}$.

Also, in Eq. (5) T and p are defined as partial derivatives of S with respect to e and v, respectively keeping all other independent variables constant. Thus, they have a strictly formal meaning in $\mathcal{G}$ but reduce to the ordinary equilibrium temperature and pressure in $\mathcal{C}$.

To first order in the nonconserved variables, substitution of (6) in (5) leads to the expression

$$T_0\,dS = p_0\,dv + de + v\alpha_{00}\mathfrak{T}\,d\mathfrak{T} + v\alpha_{10}\mathbf{q}\cdot d\mathbf{q} + v\alpha_{21}\mathring{\mathfrak{T}} : \alpha\mathring{\mathfrak{T}} \tag{7}$$

where T_0 and p_0 are the local equilibrium values of T and p, respectively. Hence, the Gibbs equation of LIT is now replaced by the one arising from (7) where the time rate of change of the nonconserved quantities remains unknown.

The next assumption of the theory is connected with the structure of the entropy balance equation. For an isotropic fluid one assumes that the entropy flux $\mathbf{J}_s$ is the most general isotropic vector in $\mathcal{G}$ so that

$$\mathbf{J}_s = \beta_0\mathbf{q} + \beta_1\mathbf{q}\cdot\mathring{\mathfrak{T}} + \beta_2\mathfrak{T}\mathbf{q} + \cdots \tag{8}$$

where

$$\beta_i = \beta_{i0} + \beta_{i1}\mathfrak{T} + \text{terms with higher-order invariants} \tag{9}$$

for $i = 0, 1, 2$. The β_{ij} are now only functions of e and ρ evaluated at equilibrium. Clearly, by comparison with LIT $\beta_{00} = T_0^{-1}$, T_0 being the local temperature. Thus, to second order in the $\mathfrak{R}$ variables,

$$\mathbf{J}_s = \frac{1}{T_0}\mathbf{q} + \beta_{01}\mathfrak{T}\mathbf{q} + \beta_{10}\mathbf{q}\cdot\mathring{\mathfrak{T}} + \cdots \tag{10}$$

If we now require that

$$\rho\frac{dS}{dt} + \operatorname{div}\mathbf{J}_s = \sigma \tag{11}$$

and make use of Eqs. (7), (10), and the conservation equations for ρ and e one arrives at the result that the entropy production is given by

$$\begin{aligned}\sigma = \mathfrak{T}&\left(\beta_{01}\operatorname{div}\mathbf{q} - \frac{1}{T_0}\operatorname{div}\mathbf{u} + \frac{1}{T_0}\alpha_{00}\frac{d\mathfrak{T}}{dt}\right)\\ &+ \mathbf{q}\cdot\left(\operatorname{grad}\frac{1}{T_0} + \frac{\alpha_{10}}{T_0}\frac{d\mathbf{q}}{dt} + \beta_{01}\operatorname{grad}\mathfrak{T} + \frac{3}{2}\beta_{10}\operatorname{div}\mathring{\mathfrak{T}}\right)\\ &+ \mathring{\mathfrak{T}} : \left[\frac{1}{T_0}(\mathring{\operatorname{grad}}\,\mathbf{u})^s + \frac{\alpha_{21}}{T_0}\frac{d\mathring{\mathfrak{T}}}{dt} + \frac{3}{2}\beta_{10}(\mathring{\operatorname{grad}}\,\mathbf{q})^s\right]\end{aligned} \tag{12}$$

Notice that the property of σ according to which in LIT it may be written as a sum of the products of forces and fluxes is lost here. Furthermore, in Eq. (12) there appear the unknown time rates of change of the $\mathfrak{R}$ variables.

The last assumption is to require now that σ is the most general scalar function in $\mathcal{G}$. This will allow a way of computing the time evolution equations for the $\mathfrak{R}$ variables. In fact,

$$\sigma = X_0\mathfrak{J} + \mathbf{X}_1 \cdot \mathbf{q} + \mathring{\mathbf{X}}_2 : \mathring{\mathfrak{J}} \tag{13}$$

where

$$\begin{aligned} X_0 &= \mu_{00} + \mu_{01}\mathfrak{J} + \mu_{02}\mathfrak{J}^2 + \mu_{03}\mathbf{q}\cdot\mathbf{q} + \mu_{04}\mathring{\mathfrak{J}}:\mathring{\mathfrak{J}} + \cdots \\ \mathbf{X}_1 &= \mu_{10}\mathbf{q} + \mu_{11}\mathfrak{J}\mathbf{q} + \mu_{12}\mathbf{q}\cdot\mathring{\mathfrak{J}} + \cdots \\ \mathring{\mathbf{X}}_2 &= \mu_{21}\mathring{\mathfrak{J}} + \mu_{22}\mathring{\mathfrak{J}}\cdot\mathring{\mathfrak{J}} + \mu_{23}(\overset{\circ}{\mathbf{q}\mathbf{q}})^s + \mu_{24}\mathfrak{J}\mathring{\mathfrak{J}} + \cdots \end{aligned} \tag{14}$$

and the μ_{ij} are functions of e and ρ evaluated in equilibrium. If σ is to be consistent with the corresponding expression in LIT we must dispose of these coefficients in (14) (such as μ_{00}) so that in the linear case, $\sigma > 0$ implies that

$$\sigma = \mu_{10}\mathbf{q}^2 + \mu_{01}\mathfrak{J}^2 + \mu_{21}(\mathring{\mathfrak{J}}:\mathring{\mathfrak{J}}) > 0 \tag{15}$$

and thus $\mu_{10} > 0$, $\mu_{01} > 0$, and $\mu_{21} > 0$.

On the other hand the assumption that $\sigma > 0$ gives no insight whatsoever on the sign of the coefficients in the various terms in (14) if second- and higher-order terms in the $\mathfrak{R}$ variables are retained. The implications of this assumption has been discussed elsewhere.(8) If we now keep those terms in σ which are of third order in the $\mathfrak{R}$ variables and we equate the two equivalent forms for σ we arrive at the sought expressions for the time rate equations of the $\mathfrak{R}$ variables, namely,(6,8)

$$\begin{aligned} \frac{d\mathfrak{J}}{dt} &= \frac{T_0}{\alpha_{00}}\left(\mu_{01}\mathfrak{J} + \frac{1}{T_0}\operatorname{div}\mathbf{u} - \beta_{01}\operatorname{div}\mathbf{q} + \mu_{02}\mathfrak{J}^2 + \mu_{03}\mathbf{q}\cdot\mathbf{q} + \mu_{04}\mathring{\mathfrak{J}}:\mathring{\mathfrak{J}}\right) \\ \frac{d\mathbf{q}}{dt} &= \frac{T_0}{\alpha_{10}}\left(\mu_{10}\mathbf{q} + \frac{1}{T_0^2}\operatorname{grad}T_0 - \beta_{01}\operatorname{grad}\mathfrak{J}\right. \\ &\qquad \left. + \frac{3}{2}\,\beta_{10}\operatorname{div}\mathring{\mathfrak{J}} + \mu_{11}\mathfrak{J}\mathbf{q} + \mu_{12}\mathbf{q}\cdot\mathring{\mathfrak{J}}\right) \\ \frac{d\mathring{\mathfrak{J}}}{dt} &= \frac{T_0}{\alpha_{21}}\left[\mu_{21}\mathring{\mathfrak{J}} + \frac{1}{T_0}(\mathring{\operatorname{grad}}\,\mathbf{u})^s - \frac{3}{2}\,\beta_{10}(\mathring{\operatorname{grad}}\,\mathbf{q})s\right. \\ &\qquad \left. + \mu_{22}(\mathring{\mathfrak{J}}:\mathring{\mathfrak{J}})^s + \mu_{23}(\overset{\circ}{\mathbf{q}\mathbf{q}})^s + \mu_{24}\mathfrak{J}\mathring{\mathfrak{J}}\right] \end{aligned} \tag{16}$$

Equations (16) are precisely of the type of the Maxwell–Cattaneo equations if we neglect all terms which contain gradients of the $\mathfrak{R}$ variables and are of order higher than 2 in such variables. For instance, θ_η

$= \alpha_{21}/T_0\mu_{21}$, etc. The relaxation times thus depend on the local values of the ordinary variables and on the nature of the system through the coefficients α_{ij} and μ_{ij}.

It is interesting to point out here that the stationary solutions of Eqs. (16) lead to the constitutive equations for $\mathbf{q}$, $\mathfrak{T}$, and $\mathring{\mathfrak{T}}$ which give rise to the Burnett equations of hydrodynamics. A discussion of this fact and its connection with the structure of hydrodynamic equations beyond the Navier–Stokes regime has been published elsewhere.(9,10)

3. DERIVATION OF EIT FROM KINETIC THEORY

Just as the results of LIT can be obtained from the Chapman–Enskog solution to the Boltzmann equation when one keeps only the first-order correction to the single-particle distribution function,(11) one would like to find out if and how the results of EIT summarized in the previous section can be derived from kinetic theory. This question has been tackled before by several authors(4,12) using the results for the various terms appearing in the entropy balance equation (11) generated by the second-order correction to the distribution function obtained from the Chapman–Enskog scheme. It is also pertinent to point out that it is at this order of approximation that one obtains the Burnett equations of hydrodynamics referred to above. Although one may indeed use the resulting expressions for $\mathbf{J}_s$ and ρs obtained in the second-order approximation of the Chapman–Enskog theory and cast them in the forms (10) and (7) as required by the EIT, the form for σ as required by (13) and thus Eq. (11) itself are not satisfied simply because the method starts from the assumption that the $f(\mathbf{r}, \mathbf{v}, t)$ is a time-independent functional of the $\mathcal{C}$ variables so that the $\mathfrak{R}$ variables never appear in the description of the states of the gas. This implies, as may be readily verified, that the relaxation equations for the $\mathfrak{R}$ variables are completely foreign to the method itself and thus one cannot attempt to derive EIT through it.

The purpose of this section is to show, however, that EIT with the Maxwell–Cattaneo equations to first order in the gradients are contained in the kinetic description of a dilute gas via the Boltzmann equation when it is solved using the 13-moment method introduced by Grad some 30 years ago.(13,14) This is not the place to go into the details of the method which is lucidly dealt with in Ref. 14, but just to remind the reader that contrary to the philosophy of the Chapman–Enskog method, in this case the single-particle distribution is expanded in terms of a complete set of functions which are taken to be n-dimensional Hermite tensor polynomials. The coefficients in this expansion are taken to depend both in space and time and satisfy a set of coupled differential equations which are obtained when the exact series for f is substituted in Boltzmann's equation. To uncouple the differential equations an arbitrary truncation is introduced, which in the very specific case of molecules interacting through a potential which goes as r^{-5} leads to a particularly simple set of closed equations for the coefficients left after the truncation. If one truncates the series in the third term and keeps only that part of the Hermite polynomial associated with it which is related to the heat flux one gets an expression for the single-particle distribution function which reads as follows:

$$f = f^{(0)}\left[1 + \frac{\overset{\circ}{\mathfrak{T}}_{\alpha\beta}}{2pRT} C_\alpha C_\beta - \frac{q_\alpha C_\alpha}{2pRT}\left(1 - \frac{C^2}{5RT}\right)\right] \tag{17}$$

Here $\mathbf{c} = \mathbf{v} - \mathbf{u}(\mathbf{r}, t)$ is the thermal velocity, p the local pressure, and R the gas constant. The temperature T_0 is defined as in the Chapman–Enskog treatment through $f^{(0)}$

$$\begin{gathered} \left\{\begin{matrix} p \\ \mathbf{u} \\ e \end{matrix}\right\} = \int f^{(0)} \left\{\begin{matrix} 1 \\ m\mathbf{v} \\ \frac{1}{2}mc^2 \end{matrix}\right\} d\mathbf{c} \\ e = \tfrac{3}{2}RT_0 \quad \text{and} \quad p = \rho RT_0 \end{gathered} \tag{18}$$

It should be emphasized at this point that f depends not only of the $\mathcal{C}$ variables through $f^{(0)}$ but also on the $\mathfrak{R}$ variables $\overset{\circ}{\mathfrak{T}}$ and $\mathbf{q}$. As it is well known in the kinetic theory of the ideal gas $\mathfrak{T}$ is not present since the bulk viscosity is equal to zero. The equations for the $\mathfrak{R}$ variables follow from the two additional equations that appear corresponding to the two nonvanishing coefficients and were derived by Grad. The corresponding relaxation times are given by

$$\theta_\eta = (\beta\rho)^{-1}, \qquad \theta_q = (\tfrac{2}{3}\beta\rho)^{1} \tag{19}$$

for Maxwell molecules only, and $\beta = (3\pi/2)(2/mK)^{1/2}A_2(5)$. K is the constant in the force law, m the mass of the molecules, and $A_2(5)$ an integral whose numerical value has been obtained in the literature.(11)

It remains to verify that the method also yields the forms required by the EIT so that the entropy balance equation is satisfied. Recalling that the Boltzmann equation formally leads to Eq. (11) if(14)

$$\begin{aligned} \rho s &= -k\int f(\ln f - 1)\, d\mathbf{c} \\ \mathbf{J}_s &= -k\int \mathbf{c}f(\ln f - 1)\, d\mathbf{c} \\ \sigma &= -k\int J(ff)\ln f\, d\mathbf{c} \end{aligned} \tag{20}$$

Then, substitution of Eq. (17) into Eqs. (20) and evaluation of the resulting integrals leads to the following results:

For the entropy per unit mass after differentiation and multiplication by T_0, one gets that

$$T_0 ds = T_0 ds^{(0)} + \frac{1}{pRT_0^2} \overset{\circ}{\mathfrak{T}} : d\overset{\circ}{\mathfrak{T}} + \frac{4}{5} \frac{1}{(\rho RT)^2} \mathbf{q} \cdot d\mathbf{q} \tag{21}$$

where $ds^{(0)}$ is the differential of the entropy for the monoatomic ideal gas. This is of the form of Eq. (7) in the particular case where the local pressure and temperature defined in Eq. (18) retain their meaning in $\mathcal{C}$ space. Equation (21) is a particular case of the generalized Pfaffian required by EIT in $\mathcal{G}$ space composed of the sum of two Pfaffians, one in $\mathcal{C}$ space and the other one in $\mathfrak{R}$ space. Notice that

$$\alpha_{10} = \frac{1}{\rho^2 RT_0}, \qquad \alpha_{10} = \frac{4}{5\rho(RT_0)^2} \tag{22}$$

For the entropy flux $\mathbf{J}_s$ we get that

$$\mathbf{J}_s = \frac{\mathbf{q}}{T_0} - \frac{4}{5} \frac{\mathbf{q} \cdot \overset{\circ}{\mathfrak{T}}}{pRT} \tag{23}$$

which once more is of the form of Eq. (10) with $\beta_{11} = -4/5pRT$.

Finally, the entropy production σ has the following form:

$$\sigma = \mu_{10} \mathbf{q} \cdot \mathbf{q} + \mu_{20} \overset{\circ}{\mathfrak{T}} : \overset{\circ}{\mathfrak{T}} \tag{24}$$

where

$$\mu_{10} = \frac{1}{(pRT)^2} \int\int d\mathbf{c}\, d\mathbf{c}'\, B(\Omega)\, d\Omega\, g\mathbf{c} \cdot \left(\frac{c^2}{5RT} - 1 \right) [\mathbf{c}, \mathbf{c}] \tag{25}$$

and a similar expression for μ_{20} in which $[\mathbf{c}, \mathbf{c}]$ is substituted by $[\overset{\circ}{\mathbf{c}\mathbf{c}}, \overset{\circ}{\mathbf{c}\mathbf{c}}]$ and the c term in the integrand is also $\overset{\circ}{\mathbf{c}\mathbf{c}}$. $B(\Omega) d\Omega$ is the differential scattering cross section and

$$[A, B] = A(\mathbf{c}') + A(\mathbf{c}_1') - A(\mathbf{c}) - A(\mathbf{c}_1)$$

$\mathbf{c}$ and $\mathbf{c}_1$ being the velocities before $\mathbf{c}^1$, $\mathbf{c}^1$ after the collision.

Thus, the Grad scheme for solving the Boltzmann equation and which in essence introduces the $\mathfrak{R}$ variables into the description of the state of the gas yields a kinetic explanation for the basic premises of EIT.

4. CONCLUDING REMARKS

Although the discussion presented in this paper has the advantage of providing some microscopic justification to a theory which has so far not found many successful practical applications(15–17) the whole story is not yet complete. In fact, there are still problems in identifying all the terms appearing in Eqs. (16) with those appearing in Grad's derivation. Curiously enough both sets of equations lead to Burnett equations(8,10) of hydrodynamics so in some way they must be equivalent.

As one ought to have expected the relaxation times arising from Grad's model are also very small for gases in absolute agreement with Maxwell's predictions. For instance, in the case of chlorine whose properties are reasonably well predicted by the Maxwell soft spheres model, taking the experimental value of the viscosity at room temperature which is $1218 \times 10^{-7} \mathrm{g/cm^{-1}\text{-}sec^{-1}}$ to evaluate $(2m/K)^{1/2}$ one gets that θ, q and θ_η are of the order 10^{-10} sec. What is then the use of a dilute gas model whose microscopic description of a theory, useless for such a system, leads to those assumptions? The answer to this question parallels that of the same one that could be posed to LIT. In fact the only microscopic model at the level of kinetic theory from which one has been able to extract the basic assumptions of LIT is the dilute gas and dilute gas mixtures using at most slightly modified versions of the Boltzmann equation. But LIT is applicable to a much broader class of systems and its validity rests mainly in the agreement between theory and experiment. Here we have a similar situation. Although we cannot claim on the absolute validity of EIT because it may be explained from Boltzmann's equation the result should give us some confidence on its postulates. It now remains to look for suitable systems in nature whose behavior requires of the introduction of additional nonconserved variables and find out if the phenomenological equations and their prediction fit within the structure of EIT. Following Maxwell's suggestion these systems ought to have state variables with non-negligible relaxation times such as viscoelastic solids, rheological fluids, polarizable media, etc. But it remains a future task to find out if EIT is a suitable structure to predict and describe their behavior.

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